Financial Contracting with Adverse Selection and Moral Hazard

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Abstract

This paper studies the problem of a bank which has to choose a contract offer to an entrepreneur in order to finance a risky investment project. The project outcome depends on the quality of the proposed project and the level of effort that the entrepreneur expends. Both quality and effort are not observable to the bank. Applying the revelation principle, the optimal contract is found by studying mechanisms which induce truthful revelation of the entrepreneur's information. The optimal contract trades off gains in expected outcome from inducing higher effort against the increasing costs of truthful revelation. It is shown that a combination of debt and equity contracts solves the contracting problem and maximizes the bank's profit. The bank proposes a menu of different combinations of debt and external equity financing, from which the entrepreneur can choose one.

1 Introduction

Most models which study the financial contracting problem between a firm and its financiers assume that the capital market is competitive and that the firm makes a take-it-or-leave-it offer to the financier.[1] As far as the contracting situation between a firm and a bank is concerned, this assumption stands in remarkable contrast to the typical real world situation. In many cases, the firm's house bank enjoys an informational advantage over its competitors and therefore can exert some monopoly power. Moreover, it is common that the bank and not the customer makes the contract offer. Since banks sell a large variety of different

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[1] For an overview, see Harris and Raviv (1992).

products, the firm will usually have a choice between different contracts, i.e. it is offered a menu of different contracts.

This paper develops a theory how a bank will design financial contracts in the presence of moral hazard and adverse selection. It studies the contracting problem between a bank and an entrepreneur who has to raise funds in order to finance a risky investment project with variable scale. The project outcome is affected by the quality of the project known only to the entrepreneur and the unobservable level of the entrepreneur's effort. Both parties are assumed to be risk neutral. The bank may try to solve the moral hazard problem by making the entrepreneur the full residual claimant of the project outcome, i.e. finance with debt only. However, it will refrain from doing so because it would have to pay high information rents to the entrepreneur in order to induce truthful revelation of the entrepreneurs type. The optimal contract is shown to trade off increases of expected outcome from inducing higher effort against increasing costs of truthful revelation. Compared to the solution under first best conditions, both the level of effort and the amount of capital investment will be distorted for all entrepreneurs except the entrepreneur with the most promising project.

It is shown that the financier achieves the maximum profit by offering a menu of different combinations of debt and equity participations to the entrepreneur. The modelled bank thus resembles the universal bank type which is engaged in both credit granting and investment banking activities.[2] An alternative view would be to interpret the financier as a venture capitalist who finances entrepreneurs with either equity or debt.

The model adds to the growing security design literature which derives financial contracts from a contractual optimization problem.[3] Instead of taking contractual forms as given, this literature endogenously derives optimal financial contracts and identifies conditions for which debt and equity are such optimal arrangements. Analogously to the security design literature, this paper derives the optimal contractual solution without making any restrictions on the type of contracts used and then shows that debt and equity contracts are sufficient to implement the optimal solution. The analysis builds on techniques that have been

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[2] Although the model assumes that the bank finances the equity participations from own funds, the results would not change when the bank acts as an underwriter and sells equity issues to the capital market.

[3] See Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Chang (1987), Williams (1989), Hart and Moore (1989), and Bolton and Scharfstein (1990).

developed in the literature on regulation under asymmetric information.[4] This literature shows that one reason for offering a menu of contracts is the response to adverse selection problems. It should be noted that debt and equity contracts are not the sole solution to the considered contracting problem. Other contractual arrangements may equivalently implement the optimal solution in the context of the model.

The paper is related both to the existing literature on adverse selection and moral hazard in financial contracting. In the literature on moral hazard situations, the financier cannot observe an action of managers. Various papers show that financial contracts can affect managerial actions, e.g. the level of effort and managerial perquisites (Jensen and Meckling (1976)), investment decisions (Jensen (1986); Stulz (1991)), the liquidation decision (Harris and Raviv (1990)), the choice of investment projects (Diamond (1989); Hirshleifer and Thakor (1989)) and the pay out of firm profits to investors (Bolton and Scharfstein (1990); Hart and Moore (1989)). Another strand of the literature is concerned with the impact of asymmetric information about characteristics of the firm on financial contracts. One part of this literature models signalling situations in which a firm's financing decisions affect the assessment of the capital market about some unknown characteristics of the firm. The key question analyzed is how firms will respond to these signalling phenomena.[5] The second kind of asymmetric information models are adverse selection models. In these models, it is not the informed party (the firm) that moves first (e.g. by making a financing decision), but instead the uninformed party takes action first. The credit rationing literature falls under this headline, where an uninformed bank has to make a contract offer taking into account the response of different types of firms.[6] The credit rationing model of Chan and Thakor (1987) is based on similar assumptions to the model presented here, i.e. they also assume adverse selection and moral hazard. However, Chan and Thakor constrain their analysis to debt contracts with different degrees of collateralization and do not analyze whether the bank could increase its profits by a richer set of contracts. Another related early model that studies both moral hazard and adverse selection in financial contracts is Baron and Holmström (1980). They consider the particular problem how the issuer should design an investment banking contract when the price of a

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[4] See Guesnerie and Laffont (1984); Laffont and Tirole (1986); Picard (1987).

[5] See Myers and Majluf (1984); Narayanan (1988); Heinkel and Zechner (1990); Brennan and Kraus (1987); Constantinides and Grundy (1989); Noe (1988).

[6] See (Jaffee and Russel (1976); Stiglitz and Weiss (1981); Wette (1983)); Chan and Thakor (1987).

security issue depends on both the effort of the investment bank and some private information of the bank.

2 The model

Consider a principal-agent model in which a bank is a principal who designs the contract governing the principal-agent relationship.[7] The bank contracts with an entrepreneur, who "owns" an investment opportunity. The investment is risky: with a probability of p, the project succeeds and yields an outcome x that depends on the level of capital investment I made by the bank, the quality of the project q and the entrepreneur's effort e. Otherwise the project fails and yields an outcome of zero. It is assumed that the project is sufficiently profitable so that all projects will be undertaken irrespective of the value of q. I and x are observable and verifiable while q and e are private information of the entrepreneur. The probability of success p is common knowledge to both parties. q will also be referred to as the type of the entrepreneur. q is distributed over an interval $\left[q, \bar{q}\right]$ with density $f(q)$ and cumulative distribution $F(q)$. As is standard in the incentive literature, the hazard rate is assumed to be monotone:[8]

$$\frac{\P \dfrac{1-F(q)}{f(q)}}{\P q} \leq 0 .$$

The outcome in case of success x is determined by

$$x \;=\; R(I,q)+e . \qquad (1)$$

The function $R(I,\theta)$ is assumed to have the following properties:

$\partial R/\partial I > 0$; $\partial^2 R/\partial I^2 < 0$,
$\partial R/\partial q > 0$; $\partial^2 R/\partial q^2 < 0$.

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[7] Although the bank is modelled as a monopolist, the model still is consistent with competition among banks, when deposits are limited and all rents accrue to depositors. See Chan and Thakor (1987).
[8] This assumption is satisfied by many classes of distributions. See Laffont and Tirole (1993) and Bagnoli and Bergström (1989).

The first line indicates that higher investment leads to higher returns, but at a decreasing rate. The second line states the same for the type *q:* better types enjoy higher returns, but again at a decreasing rate. The entrepreneur has no initial wealth and thus the bank must finance the total invested capital *I*. Without loss of generality, it is assumed that the bank receives *x* and pays a transfer $t(\hat{q},x)$ to the entrepreneur.[9] The transfer depends on the observable outcome *x* as well as a report of the entrepreneur about his type, denoted by $\hat{q}$.[10] Let $y\ (e)$ denote the disutility of effort for the entrepreneur. $\psi(e)$ is assumed to be differentiable thrice and satisfies $y' > 0,\ y'' > 0, y''' \geq 0$. For simplicity, the interest rate is normalized to zero. Assuming that both parties are risk neutral, the resulting utility functions of the bank U_B and the entrepreneur U_E for a given type *q* are:

$$U_B = p\left[R(I(\hat{q}),q)+e\right]-E\left[t(\hat{q},x)\right]-I(\hat{q}) \tag{2}$$

and

$$U_E = E\left[t(\hat{q},x)\right]-y\ (e), \tag{3}$$

where E[×] is the expectation operator. Notice that the bank can make *I* dependent on the entrepreneur's announcement $\hat{q}$. Ex ante, the bank does not know the type of the entrepreneur and her ex ante expected utility is:

$$E(U_B) = \int_{\underline{q}}^{\bar{q}} \left\{p\left[R(I(\hat{q}),q)\ +\ e\right]\ -E\left[t\,(\hat{q},x)\right]-I(\hat{q})\right\} f(q)dq \tag{4}$$

The entrepreneur has a reservation utility $\underline{U}$ of which can be thought of as the utility that the entrepreneur receives from outside opportunities. For simplicity, $\underline{U}$ is assumed to be independent of the type. However, the results do not change as long as $\underline{U}$ is not increasing too strongly with *q.*[11] His willingness to participate is expressed in the following individual rationality constraint:

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[9] Equivalently, one could assume that the entrepreneur receives the outcome and pays a transfer to the bank.

[10] The transfer payment could additionally made dependent on I. However, it will become clear that the bank cannot increase its profit from doing so.

[11] See Caillaud, Julien and Picard (1990).

$$E\left[t\left(\hat{q},x\right)\right]-y\ (e)\geq \underline{U} \qquad \text{for all } q \qquad (5)$$

3 The first best solution

Before the optimal contract under asymmetric information is analysed, the first best solution shall be determined as a benchmark for later results. When type and effort are both observable and verifiable, the bank can condition *I* and *e* on the entrepreneur's type *q* and will choose functions *I(q)* and *e(q)* such that its utility from (2) is maximized subject to the constraint that the entrepreneur receives his reservation utility from (5). Inserting the participation constraint (5) into the objective function gives:

$$U_B = p\left[R\left(I(q),q\right) + e(q)\right]-y\ \left(e(q)\right)-\underline{U}-I(q) \quad \rightarrow \quad \max_{I(q),e(q)}! \qquad (6)$$

Pointwise maximization gives the following first order conditions for an interior maximum:

$$p-\frac{¶y\ (e)}{¶e(q)}=0$$

$$p\ \frac{¶R\left(I(q),q\right)}{¶I(q)}-1=0. \qquad (7)$$

In the first best optimum, marginal return of both effort and investment equal its marginal cost. As can be easily checked, the associated second order conditions are fulfilled. The level of capital investment generally depends on the type θ: If marginal return on investment increases with θ ($\partial^2 R/\partial I\partial\theta>0$), good types will receive more capital to invest than bad types, otherwise they will receive less capital. The level of effort is independent of the entrepreneur's type, since marginal return and cost of effort do not depend on type.

4 The optimal incentive scheme

Let us now turn to the case of unobservable type and effort. As is well known from the revelation principle, the outcome of any contractual arrangement can be duplicated by a direct mechanism in which the bank makes the transfer payment dependent on an announcement of the entrepreneur about his type θ and provides the entrepreneur with incentives to reveal his type truthfully.[12] Attention can thus be restricted to incentive compatible mechanisms where the entrepreneur truthfully announces his type. While the restriction to direct mechanisms greatly simplifies the analysis, contracts of this kind are rarely found in reality. For this reason, it will later be shown in the subsequent section that the bank can alternatively use a menu of debt and equity contracts to achieve the same result.

The implementable incentive schemes are restricted by two kinds of incentive constraints: first the entrepreneur must be induced to report his type θ truthfully, second he must be given incentives to choose the desired level of effort *e**. Both constraints are summarized in the following incentive constraint:

$$(q, e^{*}) \in \arg\max_{e,\hat{q}} \left\{ E\left[t\left(\hat{q}, x\right)\right] - y\ (e) \right\} . \qquad (8)$$

When the incentive constraint (8) holds, the announcement of the true type and the level of effort *e** maximize the entrepreneurs utility. A transfer schedule $t(\hat{q}, x)$ is said to implement the effort function $e^{*}(q)$, if both the incentive constraint and the individual rationality constraint are satisfied.

The analysis proceeds in two steps: First, the set of incentive compatible schemes is determined. Subsequently, the utility maximizing scheme for the bank is selected out of the set of incentive compatible schemes. Let $x\left(\hat{q}\right)$ be the level of output that the bank requires from an entrepreneur of type $\hat{q}$ for the case that the project is successful.[13] From (1), *e* can alternatively be expressed as $e(\hat{q}) = x(\hat{q}) - R(I(\hat{q}), q)$. The entrepreneur's utility can thus be rewritten as:

$$U_{E} = E\left[t\left(\hat{q}, x(\hat{q})\right)\right] - y\ \left[x(\hat{q}) - R(I(\hat{q}), q)\right] . \qquad (9)$$

Maximization of U_E with respect to $\hat{q}$ yields the following necessary condition for truth telling:

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[12] See Fudenberg and Tirole (1991), 253 ff.

[13] It should be clear that the bank can force any desired output by severely punishing the entrepreneurs for other realizations of *x*.

$$\frac{\partial U_E}{\partial \hat{q}} = \frac{\partial E(t)}{\partial \hat{q}} - \frac{\partial y\ (e)}{\partial e}\left(\frac{\partial x}{\partial \hat{q}} - \frac{\partial R}{\partial I}\frac{\partial I}{\partial \hat{q}}\right) = 0 \qquad for\ q = \hat{q} \quad . \tag{10}$$

When $\hat{q} = q$ and $\partial U_E / \partial \hat{q} = 0$ for $\hat{q} = q$, the derivative of (9) in respect to the true type θ results to be

$$\frac{\partial U_E}{\partial q} = \frac{\partial y\ (e)}{\partial e}\frac{\partial R}{\partial q}. \tag{11}$$

This equation determines a necessary condition of how the utility of the entrepreneur changes with θ such that the entrepreneur will truthfully announce his type. Since *y '(e)* and $\partial R/\partial\theta$ are positive, U_E is increasing in *q*. It follows that the individual rationality constraint can only be binding for $\underline{q}$.

A transfer schedule *t(q,x)* implements *e(q)* only if the necessary condition is also sufficient. In order to analyse sufficiency, the incentive constraints for two arbitrary types *q* and $\hat{q}$ not to imitate each other are examined:

$$\begin{aligned} &E\left[t\left(\hat{q}, x(\hat{q})\right)\right] - y\ \left[x\left(\hat{q}\right) - R(I(\hat{q}), q)\right] \le\ E\left[t(q, x(q))\right] - y\ \left[x(q) - R(I(q), q)\right], \\ &E\left[t(q, x(q))\right] - y\ \left[x(q) - R(I(q), \hat{q})\right] \le\ E\left[t\left(\hat{q}, x(\hat{q})\right)\right] - y\ \left[x\left(\hat{q}\right) - R(I(\hat{q}), \hat{q})\right]. \end{aligned} \tag{12}$$

Adding these two inequalities gives:

$$\begin{aligned} &-y\ \left[x\left(\hat{q}\right) - R(I(\hat{q}), q)\right] + y\ \left[x(q) - R(I(q), q)\right] \\ &-y\ \left[x(q) - R(I(q), \hat{q})\right] + y\ \left[x\left(\hat{q}\right) - R(I(\hat{q}), \hat{q})\right] \le\ 0, \end{aligned}$$

which can alternatively be written as

$$\int_{q}^{\bar{q}}\int_{q}^{\hat{q}} \left\{ -y\ ''\left[x(a) - R(I(a), b)\right]\left(\frac{\partial x}{\partial a} - \frac{\partial R}{\partial I}\frac{\partial I}{\partial a}\right)\frac{\partial R}{\partial b}\right\} da\ db \le\ 0 \cdot \tag{13}$$

Since $q \le \bar{q}$, the integrand of this expression must have a non negative sign. By assumption, *y ''>0* and $\partial R/\partial q>0$. Sufficiency of the necessary condition of truth telling thus requires

$$\frac{\partial x}{\partial \theta} - \frac{\partial R}{\partial I}\frac{\partial I}{\partial \theta} \geq 0 \quad . \qquad (14)$$

The necessary and sufficient conditions for incentive compatibility are summarized in proposition 1:

PROPOSITION 1: An incentive compatible transfer schedule satisfies the following constraints:

$$\frac{\partial U_E}{\partial \theta} = \frac{\partial \psi(e)}{\partial e}\frac{\partial R}{\partial \theta},$$

$$\frac{\partial x}{\partial \theta} - \frac{\partial R}{\partial I}\frac{\partial I}{\partial \theta} \geq 0,$$

$$E\left[t(\underline{\theta}, x)\right] - \psi(e) \geq \underline{U}.$$

In order to derive the optimal contract satisfying all 3 constraints, the second constraint will be neglected for the moment and the optimal contract is derived when only the first order condition of truth telling and the participation constraint for $\underline{\theta}$ is binding. Later it will be shown that the resulting contract also fulfils constraint (14).

When both the individual rationality constraint (5) and the first order condition for truth telling (11) are binding, the transfer to an entrepreneur is

$$\psi(e(\theta)) + \underline{U} + \int_{\underline{\theta}}^{\theta} \frac{\partial \psi(e)}{\partial e}\frac{\partial R}{\partial \theta} d\theta \quad . \qquad (15)$$

The ex ante expected transfer that the bank has to pay to pay to an entrepreneur of unknown type therefore is

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \psi(e(\theta)) + \underline{U} + \int_{\underline{\theta}}^{\theta} \frac{\partial \psi(e)}{\partial e}\frac{\partial R}{\partial \theta} d\theta \right\} f(\theta) d\theta \quad . \qquad (16)$$

Integrating by parts, this expression can be rewritten as

$$\int_{\underline{q}}^{\bar{q}} \left\{ \psi\left(e(q)\right) + \underline{U} + \frac{\partial \psi(e)}{\partial e} \frac{\partial R}{\partial q} \frac{1-F(q)}{f(q)} \right\} f(q)\,dq \quad . \tag{17}$$

Inserting this payment into the objective function of the bank (4) yields:

$$E(U_B) = \int_{\underline{q}}^{\bar{q}} \left\{ p\left[R(I(q),q) + e(q)\right] - \psi\left(e(q)\right) - \underline{U} - \frac{\partial \psi(e)}{\partial e} \frac{\partial R}{\partial q} \frac{1-F(q)}{f(q)} - I(q) \right\} f(q)\,dq \to \max_{I(q),e(q)}! \tag{18}$$

The optimal contract that the bank will choose can now be characterized. Assuming that the objective function is concave,[14] pointwise maximization in respect to *e(q)* and *I(q)* gives the conditions for the choice of effort and investment. The appendix shows that these conditions also satisfy condition (14).

<u>PROPOSITION 2</u>: The optimal levels of effort and investment to be implemented by the bank are given by:

$$\frac{\partial \psi\left(e(q)\right)}{\partial e(q)} = p - \frac{\partial^2 \psi\left(e(q)\right)}{\partial e(q)^2} \frac{\partial R}{\partial q} \frac{1-F(q)}{f(q)}, \tag{19}$$

$$p\,\frac{\partial R}{\partial I(q)} - \frac{\partial \psi}{\partial e(q)} \frac{\partial^2 R}{\partial q\,\partial I(q)} \frac{1-F(q)}{f(q)} = 1 \tag{20}$$

Equation (19) determines the entrepreneur's level of effort. Compared with the first best condition (8), the first order condition under asymmetric information includes an additional term that is zero for $q = \bar{q}$ since by definition $F(\bar{q}) = 1$. The entrepreneur with the best project will therefore work as hard as under first best conditions. For $q < \bar{q}$, the additional term is positive since $\psi'(e)$ and $\partial R/\partial q$ are positive. The level of effort that the bank will enforce for the remaining entrepre-

[14] The appendix shows that the objective function is concave if $\partial^3 R/\partial^2 I \partial\theta > 0$ and the value of $\partial^2 U_B/\partial I \partial e$ is not too large.

neurs will therefore be smaller that the first best level. Equation (20) determines the optimal capital investment I. Compared to the first best condition, equation (20) again includes an additional term that is zero for $q = \bar{q}$. The entrepreneur with project $\bar{q}$ will consequently receive as much capital from the bank as he would receive under first best conditions. For $q < \bar{q}$, there is still no investment distortion if $\partial^2 R/\partial I \partial q$ equals zero, i.e. if marginal return on investment does not depend on the entrepreneur's type. Otherwise, the investment decision will be distorted as compared to the first best situation. When high quality projects deserve higher investment under first best conditions ($\partial^2 R/\partial I \partial q > 0$), there will be a downward distortion of I and the solution is characterized by underinvestment for all entrepreneurs except $\bar{q}$. If on the other hand low quality projects most urgently deserve investment, there will be an upward distortion and an overinvestment result obtains.

The result that entrepreneurs work less than under first best conditions is familiar from the traditional principal agent literature.[15] However, the underlying logic is different in the present model. In the principal agent literature, the principal trades off the benefits of higher effort against the costs in the form of higher expected compensation that results from the risk aversion of the manager. In the model presented here, the entrepreneur is risk neutral and the bank does not have to increase the expected transfer to the entrepreneur in order to induce more effort. However, the bank does not want to implement the first best level of effort for all entrepreneurs except $\bar{q}$, because increasing effort implies higher information rents that the bank has to pay in order to sustain incentive compatibility. The optimal level of effort thus trades off the impact of higher effort on the investment outcome against the increase in information rents. This can be seen most easily from the expected transfer payment to the entrepreneur in equation (17): For $q < \bar{q}$, a reduction of e reduces $y'(e)$ and thus reduces the last term in (17) which denotes the information rent of the entrepreneur. For $q = \bar{q}$, $F(q)=1$ and the information rent is zero. Hence, it cannot be further reduced by lowering e. Consider the case that the bank wanted to implement the first best effort for all types. The bank then would have to pay a strictly positive rent to all entrepreneurs except the best type $\bar{q}$. By slightly reducing the level of effort for any $q < \bar{q}$, the bank incurs only a second order loss in respect to the outcome x. However, it receives a first order gain because of the reduction of the information rents it has to pay in order to induce truthful revelation. Thus a downward distortion of the effort level is profitable for the bank. The reason why the bank will distort the level of investment rests on a similar argument: When $\partial^2 R/\partial I \partial q$ is positive, the bank can decrease $\partial R/\partial q$ by lowering I and again reduce the necessary transfer to the entrepreneur from equation (17). By giving bad types less than the first best amount of capital, it lowers the incentive of good types to imitate bad types and thereby reduces information rents. For $\partial^2 R/\partial I \partial q < 0$, overinvestment equivalently reduces the information rent paid to the entrepreneur.

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[15] See Holmström (1979), Grossman and Hart (1983).

5 Implementation with debt and equity contracts

Until now, only the conditions characterizing the optimal contract have been derived but it has not been considered how the contract might actually look like. From the revelation principle, two alternative ways for implementation are equivalent: In a direct mechanism, the bank proposes a menu of transfer schedules $t(\hat{q}, x)$ for all values of $\hat{q}$, asks the entrepreneur to announce his type q which in turn determines the actual transfer payment $t(q, x)$. In an indirect mechanism, the bank offers a menu of contracts to the entrepreneur from which the entrepreneur chooses a contract. Since direct revelation mechanisms are rarely observed in reality, the second alternative will be pursued here. The objective is to search a menu of contracts that implements the solution outlined above. In the real world, a multitude of different financial contracts exist, which in principle could all be used to achieve the bank's goals. However, the paper concentrates on the two most basic financial contracts: debt and equity contracts. A debt contract specifies a payment *D* that the entrepreneur must make to the bank. Since the entrepreneur by assumption has no own wealth and the project outcome is random, the possibility of bankruptcy must be taken into account. The bank will receive the contractual debt payments only when the outcome *x* exceeds the debt obligation *D*. Otherwise, the entrepreneur is bankrupt and the bank will seize the entrepreneur's assets and receive the outcome *x*. An equity contract specifies a linear sharing rule of the profit after debt service *x*-*D* between the bank and the entrepreneur. Let *E* be the fraction of profits that the entrepreneur obtains and (1-*E*) the fraction accruing to the bank. The bank will receive a payment from her equity participation only when profit is positive, i.e. when *x*-*D* exceeds zero. Given this contractual structure, the pay off to the entrepreneur is[16]

$$\arg\max\left(0, E(x-D)\right) = p\,E\left(x-D\right) \tag{21}$$

and the pay off to the bank is

$$\arg\max(0,(1-E)(x-D)) + \arg\min(D,x) = p(1-E)(x-D) + pD. \tag{22}$$

Let $e^*(\theta)$ and $I^*(\theta)$ be the level of effort and investment that the bank wants to implement for type θ from proposition 2. Proposition 3 shows that the bank can implement $e^*(\theta)$ and $I^*(\theta)$ by offering a menu of different external equity

[16] Note that *x* necessarily exceeds *D*. Otherwise the participation constraint of the entrepreneur cannot be satisfied.

participations and debt financing. Thus, debt and equity contracts are sufficient to achieve the optimal solution by the bank and a richer set of contracts cannot further increase the bank's profit under the model assumptions.

PROPOSITION 3: The bank can implement the optimal solution characterized in proposition 2 by offering the following contract: The entrepreneur receives a fraction

$$E = \frac{\psi'\left(e^*\left(\hat{\theta}\right)\right)}{\pi} \qquad (23)$$

of the firms profits and has to make a debt payment to the bank of

$$D = \left[R\left(I^*\left(\hat{\theta}\right),\hat{\theta}\right) + e^*\left(\hat{\theta}\right)\right] - \frac{1}{\psi'\left(e^*\left(\hat{\theta}\right)\right)}\left[\underline{U} + \psi\left(e^*\left(\hat{\theta}\right)\right) + \int_{\underline{\theta}}^{\hat{\theta}} \psi'\left(e^*\left(\tilde{\theta}\right)\right)\frac{\partial R}{\partial \tilde{\theta}}\, d\tilde{\theta}\right]. \qquad (24)$$

Proof: Three things have to be proved: First the entrepreneur must have incentives to choose the desired level of effort $e^*(\theta)$. Second, the entrepreneur must find it in his interest to pick the contract "designed for his type". Third, the entrepreneur must accept the contract. The entrepreneur will maximize expected utility from the offered set of contracts. Inserting the values for E and D into (21) and using (3) gives after rearranging terms:

$$U_E = \psi'\left(e^*\left(\hat{\theta}\right)\right)\left[R\left(I^*\left(\hat{\theta}\right),\theta\right) + e - R\left(I^*\left(\hat{\theta}\right),\hat{\theta}\right) - e^*\left(\hat{\theta}\right)\right] + \underline{u} + \psi\left(e^*\left(\hat{\theta}\right)\right) + \int_{\underline{\theta}}^{\hat{\theta}} \psi'\left(e^*\left(\tilde{\theta}\right)\right)\frac{\partial R}{\partial \tilde{\theta}}\, d\tilde{\theta} - \psi(e) \qquad (25)$$

The derivative of (25) in respect to e gives the following first order condition for the optimal level of effort chosen by the entrepreneur:

$$\frac{\partial U_E}{\partial e} = \psi'\left(e^*\left(\hat{\theta}\right)\right) - \psi'(e) = 0 \qquad (26)$$

It follows that the entrepreneur will choose the desired level of effort $e = e^*\left(\hat{\theta}\right)$. The associated second order condition is $-\psi''(e) < 0$ and has the prescribed sign. The entrepreneur will select the contract $t(\hat{\theta})$ that is designed for him, if he chooses the contract for which $\hat{\theta} = \theta$. Maximization of (25) in respect to $\hat{\theta}$ gives after some simplifications

$$\frac{\partial U_E}{\partial \hat{\theta}} = \frac{\partial^2 \psi\left(e\left(\hat{\theta}\right)\right)}{\partial \hat{\theta}^2}\left[R\left(I^*\left(\hat{\theta}\right),\theta\right) + e\left(\hat{\theta}\right) - R\left(I^*\left(\hat{\theta}\right),\hat{\theta}\right) - e^*\left(\hat{\theta}\right)\right] = 0 \quad . \qquad (27)$$

Since $e = e^*\left(\hat{\theta}\right)$ from (26), this condition implies $\hat{\theta} = \theta$. Applying the envelope theorem, the associated second order condition simplifies to $-\partial R / \partial \hat{\theta} < 0$. It follows that the necessary condition is also sufficient. Finally, the contract must not violate the participation constraint of the entrepreneur. When $e(\hat{\theta}) = e^*(\hat{\theta})$ and $\hat{\theta} = \theta$, the expected utility of the entrepreneur from of accepting the contract is

$$U_E\left(\theta\right) = \underline{U} + \int_{\underline{\theta}}^{\theta} \psi'\left(e^*\left(\tilde{\theta}\right)\right)\frac{\partial R}{\partial \tilde{\theta}}\, d\tilde{\theta} \quad . \qquad (28)$$

Since the integral is non negative, the entrepreneur receives at least $\underline{U}$ and the participation constraint (5) holds. The entrepreneur will therefore accept the contract designed for him and to choose the level of effort intended by the bank. ■

Let us now analyse the transfer schedule in detail. The equity participation of the entrepreneur is positive since $\psi'>0$. Because better types are induced to work harder, E increases in the type θ. For the best type $\bar{\theta}$, $\psi'(e)=\pi$ from (19), giving the result that $E=1$. This implies that the bank will hold no equity participation and the whole project is financed with debt. The entrepreneur is the full residual claimant and will choose the first best level of effort. Entrepreneurs with lower θ will choose a contract that includes some equity participation by the bank and will consequently work less than under first best conditions.

The formula of the debt payment D is complicated but has a straightforward interpretation. The objective of the bank is to induce effort $e^*(\theta)$ and to give the entrepreneur his reservation utility plus the required information rent. The payment D is constructed such that it gives the entrepreneur exactly this utility. Loosely speaking, the bank uses the first bracket to extract the expected profit from the entrepreneur's equity participation and uses the second bracket to give the required utility to the entrepreneur. Since both brackets have different signs, the sign of D depends on the specific parameter values. This implies that the optimal level of debt might be positive for some entrepreneurs. In this case the debt contract becomes a savings contract and can be interpreted as a salary that is paid to the entrepreneur out of the firm's returns. However, at least for $\bar{q}$, D must be negative, since $y'(e(\bar{q})) = p$ and the expected outcome $p(R+e)$ necessarily must exceed the required utility of the entrepreneur in order to make the project profitable for the bank.

6 Conclusion

Entrepreneurs that ask a bank for funding in order to finance an investment project usually find themselves confronted with a choice of different financing opportunities. Banks typically offer a variety of products to their customers and will generally demand prices that depend on the specific combination of products that the entrepreneur requests. For example, the bank may make the required interest rate on a credit dependent on the amount of credit or the amount of other business that the entrepreneur does with the bank. Furthermore, banks often limit the funds they are willing to invest in a particular customer. This limit however may depend on the product choice of the customer: he may for example receive more funds if he chooses a lower leverage. This paper has developed an explanation why banks offer such menus of contracts and developed a theory, how the bank will design its offer. The model shows that by offering a menu of contracts, the bank can make contracts contingent on the type of the entrepreneur although she cannot observe the type directly. The bank maximizes its profits by offering contracts that lead to a distortion of both the level of investment and effort as compared to the case without information asymmetrie. While the level of effort will always be below its first best level, the level of investment may turn out to be eighter higher or lower than the first best level.

The outlined model could be extended in many ways. One interesting change would be to consider alternative competitive environments. The analysis of Chan and Thakor (1987) suggest that introducing competition among banks for entrepreneurs might change the results considerably. A less radical change would be to assume an information advantage of the house bank as compared to other banks, which would lead to new (type-dependent) participation constraints.[17] Besides this, the inclusion of risk aversion by the entrepreneur would make the model more realistic. Finally, the problem could be analysed in a dynamic context, leading to such phenomena as the ratchet effect. It would be interesting to study, how the financial contracts would evolve over time in this case.

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[17] See Biglaiser and Mezzetti (1993) for an analysis of non identical competing principals.

Appendix

The second order conditions of proposition 2:
The outlined necessary conditions characterize the solution only if the objective function is concave, i.e. if its Hessian matrix is negative definite. The elements of the Hessian matrix are:

$$\frac{\partial^2 U_B}{\partial e^2} = -\frac{\partial^2 y}{\partial e^2} - \frac{\partial^3 y}{\partial e^3}\frac{\partial R}{\partial q}\frac{1-F(q)}{f(q)}, \qquad \text{(A1)}$$

$$\frac{\partial^2 U_B}{\partial I^2} = p\frac{\partial^2 R}{\partial I^2} - \frac{\partial y}{\partial e}\frac{\partial^3 R}{\partial I^2 \partial q}\frac{1-F(q)}{f(q)}, \qquad \text{(A2)}$$

and

$$\frac{\partial^2 U_B}{\partial e \partial I} = -y''(e)\frac{\partial^2 R}{\partial q \partial I}\frac{1-F(q)}{f(q)} \qquad \text{(A3)}$$

(A1) is negative given the assumptions made and (A2) is negative if it is additionally assumed that $\partial^3 R/\partial^2 I \partial q > 0$. Given this additional assumption, the Hessian matrix of both first order conditions is negative definite if its determinant is positive, i.e.

$$\frac{\partial^2 U_B}{\partial^2 e}\frac{\partial^2 U_B}{\partial I^2} - \left[\frac{\partial^2 U_B}{\partial e \partial I}\right]^2 > 0, \qquad \text{(A4)}$$

which is the case when $\partial^2 U_B/\partial e \partial I$ is not too large.

The second order condition of incentive compatibility (14) is

$$\frac{\partial x}{\partial q} - \frac{\partial R}{\partial I}\frac{\partial I}{\partial q} \geq 0.$$

Since $x(q) = R(I(q),q) + e(q)$, $\partial x/\partial q$ can be written as

$$\frac{\partial x}{\partial q} = \frac{\partial R}{\partial I}\frac{\partial I}{\partial q} + \frac{\partial R}{\partial q} + \frac{\partial e}{\partial q}. \qquad \text{(A5)}$$

Thus, (14) can be rewritten as

$$\frac{\partial e}{\partial q} \geq -\frac{\partial R}{\partial q}. \tag{A6}$$

In order to check whether (A6) holds, $\partial e / \partial q$ is determined from implicitly differentiating (19):

$$\frac{\partial e}{\partial q} = -\frac{-\frac{\partial^2 y}{\partial e^2}\frac{\partial^2 R}{\partial q^2}\frac{1-F(q)}{f(q)} - \frac{\partial^2 y}{\partial e^2}\frac{\partial R}{\partial q}\frac{\partial \frac{1-F(q)}{f(q)}}{\partial q}}{-\frac{\partial^2 y}{\partial e^2} - \frac{\partial^3 y}{\partial e^3}\frac{\partial R}{\partial q}\frac{1-F(q)}{f(q)}} > 0 \quad . \tag{A7}$$

The positive sign follow directly from the assumptions. Since $\partial e / \partial q$ and $\partial R / \partial q$ are positive, condition (A6) holds and the necessary conditions of the optimal contract from proposition 2 are also sufficient conditions.

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